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Model Update Using Modal Contribution to Static Flexibility Error

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A new technique for the parametric correction of full-order analytical stiffness matrices from reduced-order modal measurements is presented. The proposed algorithm corrects model parameters by minimizing a matrix residual formed at measurement degrees of freedom only. The residual is the difference between the measured modal contribution to the static flexibility matrix and the corresponding analytical modal contribution to the static flexibility matrix. Analytical expressions are developed for the flexibility matrix error residual gradient in terms of modal sensitivities found via Nelson's method. By utilizing a flexibility-based matrix error residual, the effect of poorly modeled inertia properties on stiffness parameter estimates is greatly reduced. Unlike techniques that utilize only static data, this technique is suitable for use on unconstrained structures. In addition, the method avoids the problems of mode selection, determining modal correspondence, and eigenvector expansion or model reduction. Numerical simulation results are presented for Kabe's model as well as a finite element model of a welded frame structure.

Introduction

OBTAINING highly accurate analytical structural models is necessary for predicting the performance of many complex systems. Examples of such systems include aircraft and spacecraft, mechanical machinery, and civil structures. To localize and quantify modeling errors or structural damage, available test data are often used to revise the analytical model. This task is commonly referred to as the model update or test/analysis correlation problem.

A large body of work has been published in the area of model update. Most of this research has centered on three model update approaches: optimal matrix update, sensitivity-based parameter update, and eigenstructure assignment. These techniques seek a refined finite element model (FEM) whose modal properties are in agreement with those from an experimental modal analysis of the structure. An overview of these techniques is provided in Ref. 1. These modal-based approaches have been widely used and successfully applied to both the model reconciliation and the damage detection problem for a variety of structures. One strength of this approach is that the modal data used by these techniques are obtained from well-established modal testing techniques. However, several difficulties remain.

One remaining problem is the difficulty in distinguishing between stiffness and inertia errors in most model update algorithms. Usually, either the measured modal data or the mass matrix is assumed to be correct. The other quantity is then adjusted to enforce mass orthogonality of the mode shapes. All remaining discrepancies between the measured data and the dynamic model are attributed to stiffness modeling errors. Because most techniques update the model to minimize a modal error residual, the stiffness parameters are simply adjusted until good agreement is obtained between modes from the model and measured modal data. However, if significant inertia modeling errors are present, the effect is to distort the values of the updated stiffness parameters from their true physical values.[2] This leads to poor predictive accuracy when the model is used to estimate quantities such as static displacements, forces, or modes that are not included in the experimental data set used to update the model.

Other difficulties associated with modal-based techniques include selecting which modes to use in the update and determining modal correspondence. For computational reasons, the number of modes must usually be kept small. However, enough modes that are sensitive to the parameters being updated must be included to achieve a well-conditioned problem. This suggests using many modes. Usually, a compromise must be made. To achieve good performance, it is also crucial that the measured modes be directly compared with the corresponding modes in the analytical model. For a complex structure, this correspondence is often difficult to determine.

A final difficulty is that usually the number of measurement degrees of freedom (DOF) is much smaller than the number of analytical DOF. Therefore, if measured mode shape data are used, these mode shapes must be expanded using an eigenvector expansion technique. Alternatively, the analytical model may be reduced. Again, several of the most commonly used expansion/reduction techniques are discussed in Ref. 1. Problems are often encountered due to errors introduced by the expansion process and from the smearing effect on modeling errors introduced by model reduction.

One way to avoid many of these difficulties is to use measured static data in the update process. Several update techniques have been developed along these lines and include those proposed by Lallement et al.,[3] Hajela and Soeiro,[4] and Sanayei and Onipede.[5] Typically, these are sensitivity-based algorithms that perform an update based on deflection shapes obtained from static testing. One serious difficulty with this approach is that static testing does not allow for free–free boundary conditions. Therefore, these techniques are unsuitable for use on unconstrained structures.

An alternative to using measured static deflection data is to use a measured static flexibility matrix obtained from dynamic test data. Recently developed techniques for fitting experimental data for complex structures have led to a computationally practical approach for obtaining estimates of normal modes and residual flexibility, which are then used to form a measured static flexibility matrix.[6–10] During this procedure, it is assumed that at least one driving point measurement is available to perform a mass normalization of the measured mode shapes. This is discussed further in Ref. 11. Alternatively, an assumed mass matrix may be used. In a recent article, the authors introduced a new method for parametric correction of full-order analytical stiffness matrices from reduced-order, dynamically measured static flexibility matrices.[12] The measured static flexibility matrix is formed using measured modal data in conjunction with a

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residual flexibility estimate. The present paper presents an alternative approach that compares model and measurement on the basis of a subspace of the static flexibility matrix, formed from flexible mode contributions. Unlike the algorithms presented in Ref. 12, no residual flexibility estimate is required and the technique is suitable for use on unconstrained structures.

The remainder of this paper presents the theoretical development of the proposed modal formulation for flexibility error reduction (ModalFLEX) algorithm. In addition, numerical simulation results are presented for the simple eight-DOF discrete model introduced by Kabe[13] and a 696-DOF FEM of an unconstrained welded-frame structure. These results demonstrate that the technique is able to localize and quantify local stiffness errors, even in the presence of significant inertia modeling errors. The results also demonstrate that the proposed algorithm is less sensitive to measurement noise than the previously developed pseudoinverse formulation (PseudoFLEX). As with the earlier approach, the proposed algorithm avoids many of the difficulties associated with mode selection, modal correspondence, and eigenvector expansion or model reduction.

Theoretical Development

Parametrization of the Stiffness Matrix

Assume we are given a static FEM of the form

$$\tilde{K}\{\tilde{q}\} = \{f\} \tag{1}$$

where $\tilde{K}$ is the structural stiffness matrix of the nominal analytical model in global coordinates, $\{\tilde{q}\}$ is a set of predicted generalized coordinates corresponding to the n global DOF, and $\{f\}$ is the set of generalized nodal forces. The problem is to find a perturbation matrix ΔK that can be used to update $\tilde{K}$ so that the model correlates with available experimental data. In other words, a new model is sought such that

$$K\{q\} = \{f\} \tag{2}$$

where the updated analytical stiffness matrix K is given by

$$K = \tilde{K} + \Delta K \tag{3}$$

and the displacements predicted by the updated model are given by $\{q\}$. It is also desired that this new model retain the basic properties of the original model such as symmetry, positive definiteness, and model connectivity. To accomplish this, the perturbation to the stiffness matrix may be parametrized as

$$\Delta K = \sum_{i=1}^{n_p} \alpha_i S_i \tag{4}$$

where n_p is the number of parameters being updated and S_i is the normalized sensitivity matrix given by

$$S_i = \tilde{p}_i \frac{\partial K}{\partial p_i} \tag{5}$$

where $\tilde{p}_i$ is the nominal value of the ith parameter. The α_i are simply the normalized changes to the updated parameter p_i given by the following ratio:

$$\alpha_i = \frac{p_i - \tilde{p}_i}{\tilde{p}_i} \tag{6}$$

In other words, the α_i indicate the percentage change of parameter p_i from the nominal model.

Flexibility Error Equation

The goal of a static, flexibility-based model update procedure is to determine the α_i of Eq. (6) that minimize the error between a measured and an analytical static flexibility matrix. This problem can be expressed as

$$\min_{\alpha} \left\| G^M - G_{mm} \right\| \tag{7}$$

where G^M is the measured flexibility matrix obtained at the measurement DOF, which are typically a small subset of the analytical DOF. Similarly, G_{mm} is the partition of the analytical flexibility matrix corresponding to the measured DOF only. Relation (7) forms the basis for a flexibility-based model update approach, which is outlined in Ref. 12. The primary issues to be resolved are the formation of G_{mm} and the choice of a solution method for relation (7). Previously, two approaches for the formation of G_{mm} were presented. The first is an explicit inverse formulation that uses a static reduction of the analytical stiffness matrix to form G_{mm}. The second is an approximation to G_{mm}, based on a pseudoinverse relationship, which avoids the static reduction and can be used with a rank-deficient stiffness matrix. Linear and nonlinear solutions were developed for both approaches. The measured static flexibility matrix G^M is constructed from the contribution of measured modes to the flexibility matrix in conjunction with a residual flexibility estimate. A procedure for obtaining the measured static flexibility estimate is outlined in several references.[6–10] It is interesting that the size of G^M is dependent only on the number of measurement DOF, not the number of modes. Therefore, there is no penalty for including more modal information in the measurement. Also, because the flexibility matrix forms the basis of comparison in these approaches, knowing the correspondence between measured and analytical mode shapes is unnecessary to perform the update. Therefore, the problems of mode selection and determining modal correspondence are avoided.

The alternative approach presented here maintains most of the advantages of the previously presented algorithm and has several additional ones. First, it does not require a residual flexibility estimate. When no residual flexibility estimate is available, the update is based on a comparison of the contribution to the static flexibility matrix for only the modes that are available in the experimental data set. Another advantage is that, because the objective function incorporates only contributions from flexible modes, rigid body effects are avoided. This makes the algorithm suited to handle both constrained and unconstrained problems. A third advantage is that this algorithm appears to be less sensitive to measurement noise than the pseudoinverse formulation previously presented. The primary disadvantage is its dependence on calculations of modal sensitivities. This makes the algorithm more computationally intensive than the previous approaches but similar in complexity to many sensitivity-based parameter update approaches currently available.

ModalFLEX

One problem facing a model update approach based on static flexibility matrices is the presence of rigid body modes when the structure is unconstrained. This poses a problem because the stiffness matrix is singular for a structure that undergoes rigid body motion. However, it is possible to define the problem only in terms of the flexible mode contribution to the static flexibility matrix. This may be written as

$$G_f = \Phi_f \Lambda_f^{-1} \Phi_f^T \tag{8}$$

where Φ_f and Λ_f are the matrix of flexible mode shapes and the diagonal matrix of corresponding eigenvalues, respectively. Although this causes no procedural change for obtaining the measured flexibility matrix from modal measurements, it does affect the way we use the analytical model. There is no longer an inverse relationship that we can exploit between G_f and the statically reduced analytical stiffness matrix. Instead, we must define the proper basis for comparing the model and measurement in this case. In Ref. 14, it is shown that the statically reduced analytical stiffness matrix can be written as

$$\bar{K} = \bar{K}_f - \bar{K}_f \Phi_{R_m} \left(\Phi_{R_m}^T \bar{K}_f \Phi_{R_m}\right)^{-1} \Phi_{R_m}^T \bar{K}_f \tag{9}$$

where $\bar{K}_f$ is simply the inverse of the flexible mode contribution to the analytical flexibility matrix with respect to measurement DOF given by

$$\bar{K}_f = \left(\Phi_{f_m} \Lambda_f^{-1} \Phi_{f_m}^T\right)^{-1} \tag{10}$$

where Φ_{f_m} contains the analytical flexible mode shapes at measurement DOF. The second term in Eq. (9) represents a rank reduction on $\bar{K}_f$ owing to the rigid body modes. The matrix Φ_{R_m} contains the analytical rigid body mode shapes at the measurement DOF.

Because $\bar{K}_f$ contains only the flexible mode contribution to the reduced stiffness matrix, we can write a new residual matrix error function for this problem as the following:

$$F(\alpha) = G_f^M - \bar{K}_f^{-1} = G_f^M - \Phi_{fm}\Lambda_f^{-1}\Phi_{fm}^T \tag{11}$$

Note that enough flexible analytical modes exist to make $\bar{K}_f$ full rank and invertible whenever the number of rigid body modes plus the number of measurement DOF is less than or equal to the number of model DOF. Because the number of model DOF is generally much larger than the number of measurement DOF, this will usually be the case.

To calculate the error function gradient, we take the partial derivative of Eq. (11) with respect to the normalized model parameters, which gives

$$\frac{\partial F}{\partial \alpha_i} = -\frac{\partial}{\partial \alpha_i}\left(\Phi_{fm}\Lambda_f^{-1}\Phi_{fm}^T\right) \tag{12}$$

and using an expression for the derivative of a matrix inverse,[15] we may write the following:

$$\frac{\partial \Lambda_f^{-1}}{\partial \alpha_i} = -\Lambda_f^{-1}\frac{\partial \Lambda_f}{\partial \alpha_i}\Lambda_f^{-1} \tag{13}$$

Expanding Eq. (12) and using Eq. (13), the gradient of the error residual with respect to a particular normalized parameter can be written as the following:

$$\frac{\partial F}{\partial \alpha_i} = -\left(\frac{\partial \Phi_{fm}}{\partial \alpha_i}\Lambda_f^{-1}\Phi_{fm}^T + \Phi_{fm}\Lambda_f^{-1}\frac{\partial \Phi_{fm}^T}{\partial \alpha_i} - \Phi_{fm}\Lambda_f^{-1}\frac{\partial \Lambda_f}{\partial \alpha_i}\Lambda_f^{-1}\Phi_f^T\right) \tag{14}$$

To use a nonlinear solution technique that requires the computation of this gradient, expressions for the modal matrix sensitivities $\partial\Phi_f/\partial\alpha_i$ and $\partial\lambda_f/\partial\alpha_i$ must be available. One way to obtain these quantities is to use Nelson's method.[16] To derive the sensitivity expressions, we start with the full-order structural eigenproblem for the jth mode given by the following equation:

$$K\{\phi_j\} = \lambda_j M\{\phi_j\} \tag{15}$$

Differentiating Eq. (15) with respect to the normalized parameter α_i and rearranging gives the following equation, which relates modal sensitivities to the mass and stiffness matrix:

$$(K - \lambda_j M)\frac{\partial}{\partial \alpha_i}\{\phi_j\} = -\left(\frac{\partial K}{\partial \alpha_i} - \lambda_j\frac{\partial M}{\partial \alpha_i} - \frac{\partial \lambda_j}{\partial \alpha_i}M\right)\{\phi_j\} \tag{16}$$

By premultiplying Eq. (16) by $\{\phi_j\}^T$ and noting that M and K are symmetric, an expression is obtained for the jth eigenvalue sensitivity:

$$\frac{\partial \lambda_j}{\partial \alpha_i} = \{\phi_j\}^T\left(\frac{\partial K}{\partial \alpha_i} - \lambda_j\frac{\partial M}{\partial \alpha_i}\right)\{\phi_j\} \tag{17}$$

Combining Eqs. (16) and (17) gives

$$(K - \lambda_j M)\frac{\partial}{\partial \alpha_i}\{\phi_j\} = \{r_j\} \tag{18}$$

where $\{r_j\}$ is given by the following:

$$\{r_j\} = -\left[\frac{\partial K}{\partial \alpha_i} - \lambda_j\frac{\partial M}{\partial \alpha_i} - \{\phi_j\}^T\left(\frac{\partial K}{\partial \alpha_i} - \lambda_j\frac{\partial M}{\partial \alpha_i}\right)\{\phi_j\}M\right]\{\phi_j\} \tag{19}$$

The complete eigenvector derivative is given by the following sum of a particular and homogeneous solution to Eq. (18):

$$\frac{\partial}{\partial \alpha_i}\{\phi_j\} = \{\nu_j\} + c_j\{\phi_j\} \tag{20}$$

The particular solution $\{\nu_j\}$ gives $\{r_j\}$ when substituted into Eq. (18). The second term is the homogeneous solution. When the mass normalization equation for the jth eigenvector given by

$$\{\phi_j\}^T M\{\phi_j\} = 1 \tag{21}$$

is differentiated with respect to α_i and combined with Eq. (20), we obtain the following expression for c_j:

$$c_j = -\{\phi_j\}^T M\{\nu_j\} - \frac{1}{2}\{\phi_j\}^T\frac{\partial M}{\partial \alpha_i}\{\phi_j\} \tag{22}$$

At first glance, it seems that the mode shape sensitivity could be solved for directly from Eq. (18). Unfortunately, the matrix $K - \lambda_j M$ is rank deficient for the distinct jth eigenvalue. Nelson's[16] solution to this problem is to set the kth term in $\{\nu_j\}$ to zero. The pivot k is chosen at the location where $|\{\phi_j\}_k|$ is a maximum. This gives the following matrix equation:

$$\begin{bmatrix} [K-\lambda_j M]_{11} & 0 & [K-\lambda_j M]_{13} \\ 0 & 1 & 0 \\ [K-\lambda_j M]_{31} & 0 & [K-\lambda_j M]_{33} \end{bmatrix}\begin{Bmatrix} \{\nu\}_1 \\ \nu_k \\ \{\nu\}_3 \end{Bmatrix} = \begin{Bmatrix} \{r\}_1 \\ 0 \\ \{r\}_3 \end{Bmatrix} \tag{23}$$

Solving Eq. (23) for $\{\nu_j\}$, the mode shape sensitivity can be formed using Eqs. (22) and (20). Partitioning $\partial/\partial\alpha_i\{\phi_j\}$ with respect to the measurement DOF, we may assemble the modal sensitivity matrices of Eq. (14) as

$$\frac{\partial \Phi_{fm}}{\partial \alpha_i} = \left[\frac{\partial\{\phi_1\}_m}{\partial \alpha_i} \quad \frac{\partial\{\phi_2\}_m}{\partial \alpha_i} \quad \cdots \quad \frac{\partial\{\phi_{n_f}\}_m}{\partial \alpha_i}\right] \tag{24}$$

$$\frac{\partial \Lambda_f}{\partial \alpha_i} = \mathrm{diag}\left(\frac{\partial \lambda_j}{\partial \alpha_i}\right) \qquad j = 1, \ldots, n_f \tag{25}$$

where n_f is the number of flexible modes.

Using these modal sensitivity matrices, Eqs. (11) and (14) may be used to implement a nonlinear gradient search for the unknown parameters, as was done with the techniques presented in Ref. 12. Alternatively, a multivariable nonlinear optimization routine could be implemented that either uses no gradient information or utilizes finite difference approximations for calculating the gradient. These alternative approaches were not investigated in this work. Like all sensitivity-based methods that require the solution of nonlinear equations, there is the issue of solution uniqueness and dependence on the initial guess. Although the issue has not been investigated extensively, experience suggests that the solution tends to converge to the global minimum even for relatively large parameter changes. Another issue is that Nelson's[16] method cannot be used for the case of repeated eigenvalues. In this case there are several other methods that may be utilized.[17–20]

Numerical Simulation Results

Kabe's Eight-DOF Mass–Spring Model

These simulations were conducted using the eight-DOF example presented by Kabe.[13] This model includes 8 masses and 14 springs with a connectivity, as shown in Fig. 1. The model is characterized by large relative differences in mass and stiffness, which lead to poor numerical conditioning and closely spaced modes. The mass and stiffness parameters are shown in Table 1. In Kabe's original problem, it is assumed that the mass values are the same in both the analytical and experimental models. For the present work, a set of perturbed model mass parameters is also included in Table 1. These are used in some of the simulations to generate inertia modeling errors in addition to the stiffness modeling errors of the original problem.

The mass and stiffness matrix of the experimental model are formed using the exact mass and stiffness values. The matrix is then used to generate the complete set of experimental frequencies and mass-orthonormal mode shapes. The "data" for each simulation consist of an experimental modal set, which is taken to be a subset of the complete experimental modal set, starting with the lowest-frequency mode. The rest of the mode shapes and frequencies are considered to be unmeasured residual modes.

Stiffness and mass modeling errors are simulated by replacing exact values with model values to form the analytical mass and stiffness matrices. Normalized sensitivity matrices for the 14-spring stiffness parameters are computed using Eq. (5).

Updating All Stiffnesses with Perfect Full-Order Measurement

For this simulation, it is assumed that perfect measurements are available at all eight DOF. When there is no noise and all DOF are measured, an exact update of all 14 spring stiffnesses is achieved when 3 or more measured modes are used. As noted by Kabe,[13] this is the minimum number of modes necessary to uniquely determine the solution of this problem. Fewer modes are needed when fewer parameters are updated.

Updating All Stiffnesses in Presence of Inertia Errors with Perfect Measurement

For this simulation, significant inertia modeling errors are introduced by utilizing the model mass values in the analytical mass matrix. The update is then performed as in the preceding, with the results shown in Table 2. Despite the large errors in mass parameters, the algorithm still converges to the exact stiffness parameter estimate when all eight modes are included in the update. In other words, inertia modeling errors had no effect on the stiffness parameter update, even with some of the mass values having errors as high as 100%. When only three modes are used, the update results in significant parameter errors. However, these errors are not uniform. Many of the parameters are very accurately estimated. Upon further investigation, one finds that the parameters with significant error are those with very little sensitivity to the measurement. It appears that these errors are the artifact of the extreme ill-conditioning of Kabe's problem.[13]

Detecting Errors in Interior Elements with Reduced-Order Measurement

Whereas the preceding case illustrates that exact updates are achieved when all model DOF are measured, another interesting aspect is the ability of the algorithm to detect errors in interior elements using a reduced-order measurement. This is because the number of measurement DOF is typically much smaller than the number of model DOF in applications of interest. So for these simulations, a challenging test case is formed by assuming that only the two distant DOF associated with m_1 and m_8 are measured and three interior spring stiffnesses are to be updated. Figure 2 shows the location of the measurement DOF and the springs containing errors. Except for the updated springs, the model uses exact stiffness values. Because only two DOF are measured, the experimental modal vectors contain only two elements. This results in a 2×2 symmetric error matrix. Because there are only three independent measurements in this case, at most three parameters may be uniquely determined. Note that, in most practical problems, the number of measurement DOF will be sufficiently high to estimate the desired number of parameters.

The results for this test case are shown in Table 3. The solution converges to the exact result in just a few iterations. These results demonstrate that it is possible to both localize and quantify model error involving relatively large parameter changes in an interior region located away from the measurement DOF.

Effect of Measurement Noise

A final set of simulations was conducted to examine the sensitivity of the proposed algorithm to measurement noise. To facilitate comparison with previously developed techniques, the same procedure was used as that outlined in Ref. 12. First, it is assumed that all modes are available and measurements are obtained at all DOF of

Table 1 Mass and stiffness parameters for Kabe's[13] problem

	Mass values			Stiffness values		Stiffness
Mass	Exact	Model	Spring	Exact	Model	error (%)
m_1	0.001	0.002	k_1	1.5	2.0	33.3
m_2	1.000	0.750	k_2	10.0	10.0	0.0
m_3	1.000	1.100	k_3	100.0	200.0	100.0
m_4	1.000	1.200	k_4	100.0	200.0	100.0
m_5	1.000	2.000	k_5	100.0	200.0	100.0
m_6	1.000	1.000	k_6	10.0	10.0	0.0
m_7	1.000	0.500	k_7	2.0	4.0	100.0
m_8	0.002	0.001	k_8	1.5	2.0	33.3
			k_9	1000.0	1500.0	50.0
			k_{10}	900.0	450.0	−50.0
			k_{11}	1000.0	1500.0	50.0
			k_{12}	1000.0	1500.0	50.0
			k_{13}	900.0	450.0	−50.0
			k_{14}	1000.0	1500.0	50.0

Table 2 Exact update achieved by ModalFLEX in presence of inertia modeling errors

Normalized	Exact	ModalFLEX solution	
parameter	solution	Eight modes	Three modes
α_1	−0.25	−0.25	−0.26
α_2	0.00	0.00	1.00
α_3	−0.50	−0.50	−0.50
α_4	−0.50	−0.50	−0.50
α_5	−0.50	−0.50	−0.50
α_6	0.00	0.00	0.10
α_7	−0.50	−0.50	0.05
α_8	−0.25	−0.25	0.52
α_9	−0.33	−0.33	0.27
α_{10}	1.00	1.00	1.00
α_{11}	−0.33	−0.33	−0.34
α_{12}	−0.33	−0.33	−0.34
α_{13}	1.00	1.00	1.00
α_{14}	−0.33	−0.33	0.09

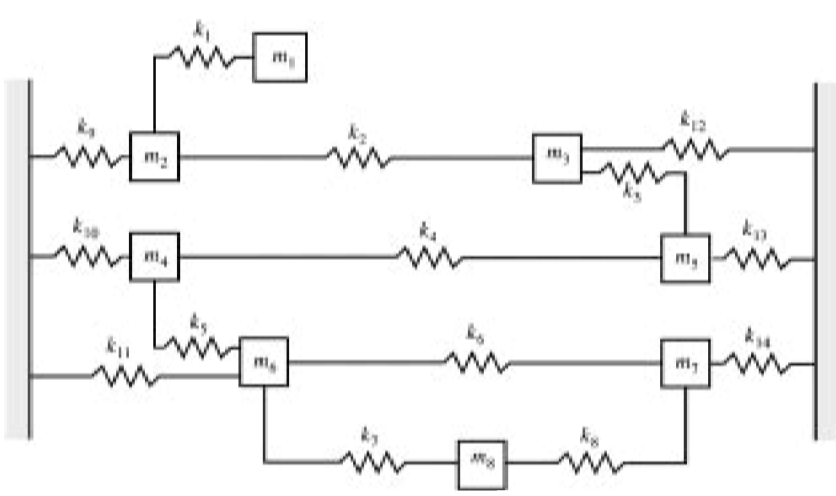

Fig. 1 Eight-DOF Kabe[13] model.

the model. Measurement noise is simulated by adding white noise to each of the simulated measured mode shapes. The maximum magnitude of the noise vector coefficients is set to be the product of a noise percentage factor with the Euclidean norm of the mass-normalized mode shape. The noise percentage factor remains the same for each mode during a given simulation.

The effect of measurement noise is studied by comparing the relative error in the updated stiffness values to the relative error in the mass-normalized measured modal matrix. The relative error in the updated stiffness values is expressed in terms of the normalized parameters as

$$\frac{\|\{p\}_{\text{final}} - \{p\}_{\text{exact}}\|_2}{\|\{p\}_{\text{exact}}\|_2} = \frac{\|\{\alpha\}_{\text{final}} - \{\alpha\}_{\text{exact}}\|_2}{\|\{1\} + \{\alpha\}_{\text{exact}}\|_2} \tag{26}$$

and the relative error in the mass-normalized modal matrix is given by

$$\frac{\|\Delta\Phi\|_2}{\|\Phi_{\text{exact}}\|_2} \tag{27}$$

where $\Delta\Phi$ is simply the matrix whose columns are the white noise vectors that were added to the individual mode shapes. For each algorithm, three update cases are studied. The first is the update of all 14 spring stiffness values. The second is the update of α_1, α_7, and α_8 only. The final case studied is the update of α_1 and α_7 only. The parameters in the last two cases were chosen because the measurements were relatively sensitive to these parameters. Several simulations with various amounts of measurement noise were conducted for each test case. The simulation results for the ModalFLEX algorithm are shown in Fig. 3. For comparison, the results for the PseudoFLEX algorithm, which were published in Ref. 12, are presented in Fig. 4.

These results show that the ModalFLEX algorithm is less sensitive to measurement noise than the previously developed PseudoFLEX algorithm. However, sensitivity to measurement is still a serious issue, particularly when all 14 parameters are estimated. This deserves several comments. First, the parameter error is concentrated in the parameters for which the error residual had little sensitivity. Note that the full update of Kabe's[13] problem is severely ill-conditioned, with the stiffness and mass values varying by over three orders of magnitude. In practice, parameters for which the observations have virtually no sensitivity would not be estimated. The results for the two- and three-parameter update cases demonstrate that the performance is improved as the problem becomes better conditioned. In contrast to the PseudoFLEX approach, ModalFLEX actually attenuates the error in the two-parameter case. Another factor is that these simulations assume that the measurement noise is uniform for all of the measured modes. In practice, lower-frequency modes may have less error owing to measurement noise than high-frequency modes. Because the weighting in this approach emphasizes contributions from lower-frequency modes, it should be less affected by the relatively higher errors that may exist in high-frequency modal measurements.

Table 3 Exact update achieved by ModalFLEX using reduced-order measurement

Normalized parameter	Exact solution	ModalFLEX solution: Eight modes	ModalFLEX solution: Three modes
α_4	−0.50	−0.50	−0.50
α_{10}	1.00	1.00	1.00
α_{13}	1.00	1.00	1.00

Welded-Frame Structure

Simulations were also conducted using a 696-DOF NASTRAN model of a three-dimensional unconstrained welded-frame structure. The geometry of the structure is illustrated in Fig. 5. The rails and cross members of the structure are modeled with beam

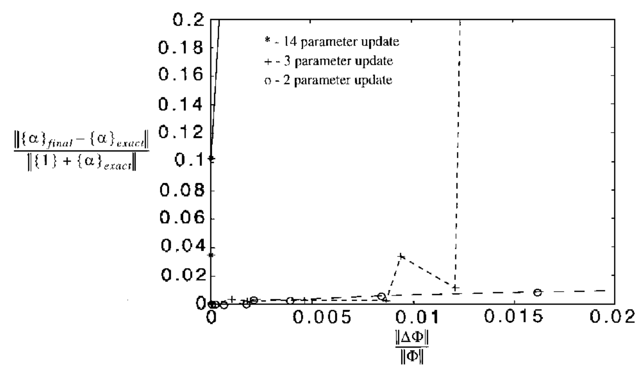

Fig. 3 Effect of measurement noise on Kabe's[13] problem using ModalFLEX.

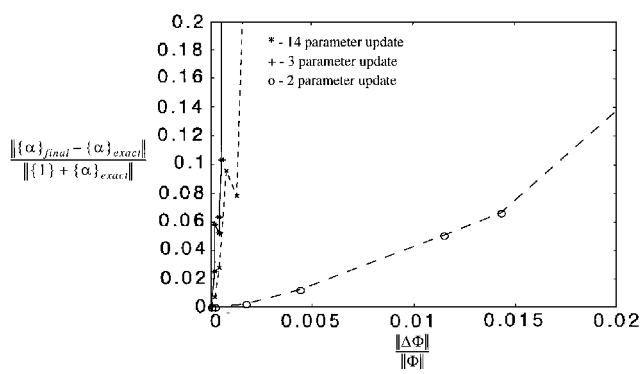

Fig. 4 Effect of measurement noise on Kabe's[13] problem using PseudoFLEX.

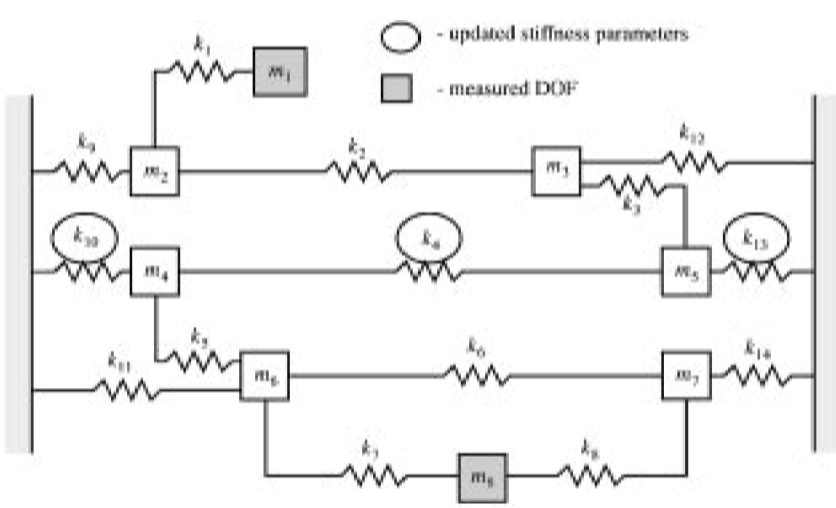

Fig. 2 Measurement and model error locations for interior spring update.

Table 4 Initial parameter values for welded-frame structure model

Parameter	Value	Normalized parameter
k_1 (N/mm)	6.01×10^6	α_1
k_2 (N/mm)	6.01×10^6	α_2
k_3 (N/mm)	6.01×10^6	α_3
k_4 (N · mm/rad)	2.53×10^8	α_4
k_5 (N · mm/rad)	1.96×10^8	α_5
k_6 (N · mm/rad)	4.80×10^8	α_6
E (N/mm^2)	2.07×10^6	α_7
I_{zz} (mm^4)	5.89×10^5	α_8
I_{yy} (mm^4)	3.11×10^5	α_9
J (mm^4)	6.15×10^5	α_{10}
A (mm^2)	7.37×10^2	α_{11}
G (N/mm^2)	8.09×10^5	α_{12}

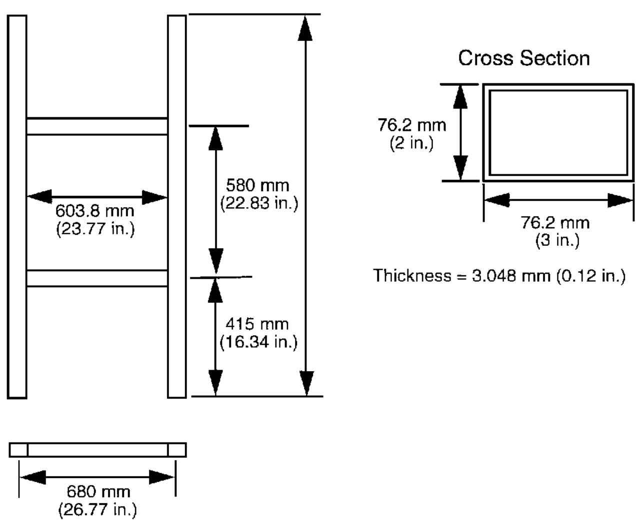

Fig. 5 Geometry for welded-frame structure.

elements. Each joint is modeled using a combination of a beam element and six general spring elements connecting each corresponding DOF. The first three general springs are between corresponding linear displacements, and the second three are between the rotational DOF. The initial model parameter values can be found in Table 4. Using NASTRAN, the normalized sensitivity matrices were obtained for several of these model parameters. The normalized parameters α_i are also shown in Table 4, alongside their respective physical parameters.

For this simulation, it is assumed that measurements can be obtained at a 96-DOF subset of the full 696 analytical DOF. It is assumed that the mass matrix is perfect and remains unchanged. Modeling errors are simulated by changing three of the model parameters. These were chosen to introduce both local and global modeling errors. Because the greatest uncertainty is associated with the joints, the first rotational spring stiffness of the joints, k_4, was decreased by 50% and the second rotational spring stiffness of the joints, k_5, was increased by 25%. The translational spring stiffnesses were not included because the measurements are extremely insensitive to these parameters. A 10% decrease in Young's modulus was also introduced to provide a global modeling error. Note that a variety of techniques exist for parameter selection. Typically, parameters are selected on the basis of their expected uncertainty as well as measurement sensitivity to these parameters. Although an examination of various parameter selection techniques was not performed in this work, parameter selection should be carefully considered before application to experimental data.

Table 5 shows the results for this simulation, which were achieved using the first 14 modes. The exact parameter values were achieved after a few iterations.

Table 5 Exact update achieved for welded-frame structure parameters using reduced-order measurement and 14 modes

Parameter	Exact solution	ModalFLEX solution (96 DOF)
α_4	1.00	1.00
α_5	−0.20	−0.20
α_7	0.11	0.11

Conclusion

This paper presents a new technique for the parametric correction of full-order analytical stiffness matrices from reduced-order modal measurements. The ModalFLEX algorithm corrects model parameters by minimizing a matrix residual formed at measurement DOF only. The residual is the difference between the measured modal contribution to the static flexibility matrix and the corresponding analytical modal contribution to the static flexibility matrix. Analytical expressions were developed for the flexibility matrix error residual gradient in terms of modal sensitivities found via Nelson's[16] method. Using these analytical modal sensitivities, this approach was implemented as a nonlinear gradient search.

Simulation results on Kabe's[13] problem demonstrate that exact updates of the stiffness parameters are achieved, even in cases where there are significant inertia modeling errors. Simulation results also show that the method can detect and quantify local model errors in elements that do not touch the measurement DOF. Additional simulations have shown that the ModalFLEX algorithm is less sensitive to measurement noise than the previously developed Pseudo-FLEX technique. Simulations involving the 696-DOF model of a welded-frame structure show that the technique is also applicable to a large-order, complex structure.

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